Integrales

$\int \frac{du}{\sqrt{a^2-u^2}} = \text{sen}^{-1}\frac{u}{a} + c$
$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{u-a}{u+a}\right\rvert + c$
$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left\lvert\frac{a+u}{a-u}\right\rvert + c$
$\int \frac{du}{a^2-u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{\lvert u\rvert}{a} + c$
$\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$
$\int \frac{-1}{1+x^2}dx = \cot^{-1}x + c$

$\int dx = x+c$ $\quad$ $\int k\,dx = kx+c$	$\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)dx$
$\int (1/x)\,dx = \ln \lvert x\rvert + c$	$\int e^x dx = e^x + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1)$	$\int a^x dx = \frac{a^x}{\ln a} + c$
$\int \cos x\,dx = \text{sen}\,x + c$	$\int \text{sen}\,x\,dx = -\cos x + c$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert + c$	$\int \cot x\,dx = \ln\lvert\text{sen}\,x\rvert + c$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$
$\int \sec x \tan x\,dx = \sec x + c$	$\int \csc x \cot x\,dx = -\csc x + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \sec^2 x\,dx = \tan x + c$
$\int (1+\tan^2 x)\,dx = \tan x + c$	$\int \cos^n u\ \text{sen}\,u\,du = \frac{\cos^{n+1}u}{n+1} + c$
$\int \frac{1}{\cos^2 x}dx = \tan x + c$	$\int \frac{-1}{\text{sen}^2 x}dx = \cot x + c$
$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\text{sen}2u + c$	$\int \text{sen}^2 u\,du = \frac{1}{2}u - \frac{1}{4}\text{sen}2u + c$
$\int \frac{-1}{\sqrt{1-x^2}}dx = \cos^{-1}x + c$	$\int \frac{1}{\sqrt{1-x^2}}dx = \text{sen}^{-1}x + c$

Transformada de Laplace

Propiedades

$$L\{a\,f(t) + b\,g(t)\} = aL\{f(t)\} + bL\{g(t)\}$$

$$L\{f'(t)\} = sL\{f(t)\} - f(0)$$

$$L\{f''(t)\} = s^2 L\{f(t)\} - sf(0) - f'(0)$$

$$L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1}f(0) - \dots - f^{n-1}(0) = s^n L\{f(t)\} - \sum_{i-1}^{n} s^{n-i} f^{(i-1)}(0)$$

$$L\{\int_a^t f(t)dt\} = \frac{F(s)}{s} + \frac{\left[\int_a^t f(t)dt\right]_{t=0}}{s}$$

$$L\{t\,f(t)\} = -F'(s)$$

$$L\{e^{at} f(t)\} = F(s-a)$$

$$L\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$$

$$L\{t^n f(t)\} = (-1)^n D_s^n [F(s)]$$

Transformada Z

Propiedades

$$Z\{f(t)\} = Z\{f(KT)\} = f(z) = \sum_{K=0}^{\infty} f(KT)\,z^{-k}$$

$$Z\{a\,x(t)\} = a\,Z\{x(t)\} \quad Z\{a^k x(KT)\} = x(a^{-1}z)$$

$$Z\{a\,x(t) + b\,y(t)\} = a\,Z\{x(t)\} + b\,Z\{y(t)\}$$

$$Z\{x(t-nT)\} = z^{-n}x(z)$$

$$Z\{e^{-at}x(t)\} = x(e^{aT}z)$$

a^k	$\frac{z}{z-a}$
$k\,a^{k-1}$	$\frac{z}{(z-a)^2}$
$\frac{1}{2}k(k-1)a^{k-2}$	$\frac{z}{(z-a)^3}$
$\frac{1}{(M-1)!}\left[\prod_{i=0}^{M-2}(k-i)\right]a^{k-M+1}$	$\frac{z}{(z-a)^M}$

Dominio s	Dominio t	Dominio z
1	$\delta(t)$	1
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{1}{2}t^2$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{(m-1)!}{s^m}$	t^{m-1} m=1,2,...	$\lim_{b\to 0}\left[(-1)^{m-1}\frac{\partial^{m-1}\frac{z}{z-e^{-bT}}}{\partial b^{m-1}}\right]$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-bt}-e^{-at}}{a-b}$	$\frac{1}{a-b}\left[\frac{z}{(z-e^{-bT})} - \frac{z}{(z-e^{-aT})}\right]$
$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tz\,e^{-aT}}{(z-e^{-aT})^2}$
$\frac{a}{s^2(s+a)}$	$t - \frac{1-e^{-at}}{a}$	$\frac{Tz}{(z-1)^2} - \frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
$\frac{a}{s^2+a^2}$	sen (at)	$\frac{z\,\text{sen}(aT)}{z^2 - 2z\cos(aT) + 1}$
$\frac{s}{s^2+a^2}$	cos (at)	$\frac{z(z-\cos(aT))}{z^2 - 2z\cos(aT) + 1}$
$\frac{b}{(s+a)^2+b^2}$	$\frac{1}{b}e^{-at}\text{sen}\,bt$	$\frac{1}{b}\left[\frac{ze^{-aT}\text{sen}\,bT}{z^2 - 2ze^{-aT}\cos(bT) + e^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$	$\frac{z^2 - ze^{-aT}\cos(bT)}{z^2 - 2ze^{-aT}\cos(bT) + e^{-2aT}}$
$\frac{\cos\theta(s+a) + \omega\,\text{sen}\theta}{(s+a)^2+\omega^2}$	$e^{-at}\cos(at-\theta)$	$\frac{z\cos\theta(z-\alpha) - z\beta\,\text{sen}\theta}{(z-\alpha)^2+\beta^2}$ $\alpha = e^{-aT}\cos\omega T$ $\beta = e^{-aT}\text{sen}\,\omega T$

Derivadas

$\frac{d}{dx}[cu] = cu'$	$\frac{d}{dx}[u \pm v] = u' \pm v'$
$\frac{d}{dx}[uv] = uv' + vu'$	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' + uv'}{v^2}$
$\frac{d}{dx}[c] = 0$	$\frac{d}{dx}[u^n] = nu^{n-1}u'$
$\frac{d}{dx}[x] = 1$	$\frac{d}{dx}[\lvert u\rvert] = \frac{u}{\lvert u\rvert}(u'),\ u \neq 0$
$\frac{d}{dx}[\ln u] = \frac{u'}{u}$	$\frac{d}{dx}[e^u] = e^u u'$
$\frac{d}{dx}[\text{sen}\,u] = (\cos u)u'$	$\frac{d}{dx}[\cos u] = -(\text{sen}\,u)u'$
$\frac{d}{dx}[\tan u] = (\sec^2 u)u'$	$\frac{d}{dx}[\cot u] = -(\csc u^2)u'$
$\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$	$\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
$\frac{d}{dx}[\text{sen}^{-1}u] = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}[\cos^{-1}u] = \frac{-u'}{\sqrt{1-u^2}}$
$\frac{d}{dx}[\tan^{-1}u] = \frac{u'}{1+u^2}$	$\frac{d}{dx}[\cot^{-1}u] = \frac{-u'}{1+u^2}$
$\frac{d}{dx}[\sec^{-1}u] = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}[\csc^{-1}u] = \frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$

Dominio s	Dominio t	Dominio z
Teorema del valor inicial		
$\lim_{s\to\infty} sF(s)$	$f(0)$	$\lim_{z\to\infty} F(z)$ si el límite existe
Teorema del valor final		
$\lim_{s\to 0} sF(s)$	$f(\infty)$	$\lim_{z\to 1}(z-1)F(z)$ si todos los polos de $(1-z^{-1})F(z)$ se ubican en $\lvert z\rvert < 1$

Propiedad de:

Trigonometría

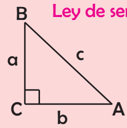
	0°	30°	45°	60°	90°
Sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
Cot	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
Sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞
Csc	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

$$\text{sen}(x \pm y) = \text{sen}\,x \cos y \pm \cos x\,\text{sen}\,y$$
$$\cos(x \pm y) = \cos x \cos y \mp \text{sen}\,x\,\text{sen}\,y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$
$$\sec(x \pm y) = \frac{1}{\cos(x \pm y)}$$
$$\cot(x \pm y) = \frac{1}{\text{sen}(x \pm y)}$$

$$\text{sen}^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\cos^2 x + \text{sen}^2 x = 1$$
$$\sec^2 x - \tan^2 x = 1$$
$$\csc^2 x - \cot^2 x = 1$$

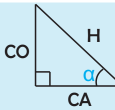
Ley de senos



$$\frac{a}{\text{Sen } A} = \frac{b}{\text{Sen } B} = \frac{c}{\text{Sen } C}$$

Ley de cosenos

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



$$\text{Sen } \alpha = \frac{CO}{H} \quad \text{Cos } \alpha = \frac{CA}{H} \quad \text{Tan } \alpha = \frac{CO}{CA}$$
$$\text{Cot } \alpha = \frac{CA}{CO} \quad \text{Sec } \alpha = \frac{H}{CA} \quad \text{Csc } \alpha = \frac{H}{CO}$$

Grados	Radianes
360°	2π rad
270°	3π/2 rad
180°	π rad
90°	π/2 rad
60°	π/3 rad
45°	π/4 rad
30°	π/6 rad
57.3°	1 rad

Grados → Radianes: Multiplicar por $\frac{180}{\pi}$

Radianes → Grados: Multiplicar por $\frac{\pi}{180}$

Razones trigonométricas

$$\tan\theta = \frac{\text{sen}\,\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\text{sen}\,\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\text{sen}\,\theta}$$

Ley de tangentes

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} \qquad \frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)} \qquad \frac{a-c}{a+c} = \frac{\tan\left(\frac{A-C}{2}\right)}{\tan\left(\frac{A+C}{2}\right)}$$

Álgebra

Euler

$$e^{j\theta} = \cos\theta + j\,\text{sen}\,\theta$$
$$e^{-j\theta} = \cos\theta - j\,\text{sen}\,\theta$$
$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$
$$e^{j\theta} - e^{-j\theta} = 2j\,\text{sen}\,\theta$$
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\text{sen}(wt) = \frac{e^{jwt} + e^{-jwt}}{2j}$$

$$(a^m)(a^n) = a^{m+n}$$
$$(ab)^m = a^m b^m$$
$$(a^m)^n = a^{mn}$$
$$a^{m/n} = \sqrt[n]{a^m}$$
$$a^0 = 1$$
$$\frac{a^m}{a^n} = a^{m-n}$$

Propiedades de los logaritmos

$$\log_a a^x = x \qquad \log_a 1 = 0$$
$$a^{\log_a x} = x \qquad \log_a a = 1$$
$$\log_a (u \cdot v) = \log_a u + \log_a v$$
$$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$$
$$\log_a (u^n) = n \log_a u$$
$$\log_a \sqrt[n]{u} = \frac{1}{n} \log_a u$$

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$$
$$2(a^2 + b^2) = (a+b)^2 + (a-b)^2$$
$$(a+b)^2 - (a-b)^2 = 4ab$$
$$a^4 + b^4 = (a+b)(a-b)[(a+b)^2 - 2ab]$$
$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$
$$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$
$$a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 - a + 1)$$
$$a^8 - b^8 = (a^4 + b^4)(a^2 + b^2)(a+b)(a-b)$$

Fórmula general

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Geometría analítica del espacio

$$P_1 = (x_1, y_1, z_1), \; P_2 = (x_2, y_2, z_2)$$
$$\overline{P_1P_2} = \{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\} = (l, m, n)$$
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} = \sqrt{l^2 + m^2 + n^2}$$

Recta que pasa por dos puntos

Forma paramétrica

$$x = x_1 + lt \qquad y = y_1 + mt \qquad z = z_1 + nt$$

Forma simétrica

$$t = \frac{x - x_1}{l} \qquad t = \frac{y - y_1}{m} \qquad t = \frac{z - z_1}{n}$$

cosenos directores

$$\cos\alpha = \frac{x_2 - x_1}{d} = \frac{l}{d} \qquad \cos\beta = \frac{y_2 - y_1}{d} = \frac{m}{d} \qquad \cos\gamma = \frac{z_2 - z_1}{d} = \frac{n}{d}$$

donde α, β y γ denotan los ángulos que forman la línea que une los puntos P1 y P2 con la parte positiva de los ejes x, y, z respectivamente

Vectores

$$\mathbf{A \cdot B} = ||\mathbf{A}||\,||\mathbf{B}|| \cos\theta \qquad 0 < \theta < \pi$$

θ ángulo formado por A y B

$$\mathbf{A \cdot B} = A1\,B1 + A2\,B2 + A3B3$$

donde $A = A_1\hat{\imath} + A_2\hat{\jmath} + A_3 k$, $B = B_1\hat{\imath} + B_2\hat{\jmath} + B_3 k$

Laplaciano

$$U = \nabla^2 U = \nabla \cdot (\nabla U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$||\mathbf{A \times B}|| = ||\mathbf{A}||\,||\mathbf{B}|| \,\text{sen}\,\theta$$

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$A \times B = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A1 & A2 & A3 \\ B1 & B2 & B3 \end{vmatrix} = (A2B3 - A3B2)\,\hat{\imath} + (A3B1 - A1B3)\,\hat{\jmath} + (A1B2 - A2B1)\,\hat{k}$$

$$\text{div } \mathbf{A} = \nabla \cdot \mathbf{A} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(A1\,\hat{\imath} + A2\,\hat{\jmath} + A3\,\hat{k}\right)$$

A=A(x,y,z) tiene derivadas parciales

$$\text{rot } \mathbf{A} = \nabla \times \mathbf{A} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(A1\,\hat{\imath} + A2\,\hat{\jmath} + A3\,\hat{k}\right)$$
$$\left(\frac{\partial A3}{\partial y} - \frac{\partial A2}{\partial z}\right)\hat{\imath} + \left(\frac{\partial A1}{\partial z} - \frac{\partial A3}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial A2}{\partial x} - \frac{\partial A1}{\partial y}\right)\hat{k}$$

A=A(x,y,z) tiene derivadas parciales

$$\text{grad } U = \nabla U = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) U = \hat{\imath}\frac{\partial U}{\partial x} + \hat{\jmath}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}$$

U=U(x,y,z) tiene derivadas parciales